

Photon dissociation into two and three jets:
initial and final state corrections *

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We study the impact factor for the photon to quark, antiquark and gluon transition within Balitsky's shock-wave formalism. Our aim is to extend existing results beyond approximations discussed in the literature. We present our results of the real contribution, and present some intermediate results on virtual contributions for the photon to quark, antiquark transition.

1. Introduction

Among the achievements of HERA, one of the major results was the experimental evidence [1, 2] that among the whole set of $\gamma^*p \rightarrow X$ deep inelastic scattering events, almost 10% are diffractive (DDIS), of the form $\gamma^*p \rightarrow XY$ with a rapidity gap between the proton remnants Y and the hadrons X coming from the fragmentation region of the initial virtual photon.

There are two main approaches to theoretically describe diffraction. The first one involves a *resolved* Pomeron contribution, see Fig. 1 (left), while the second one relies on a *direct* Pomeron contribution involving the coupling of a Pomeron with the diffractive state, see Fig. 1 (right).

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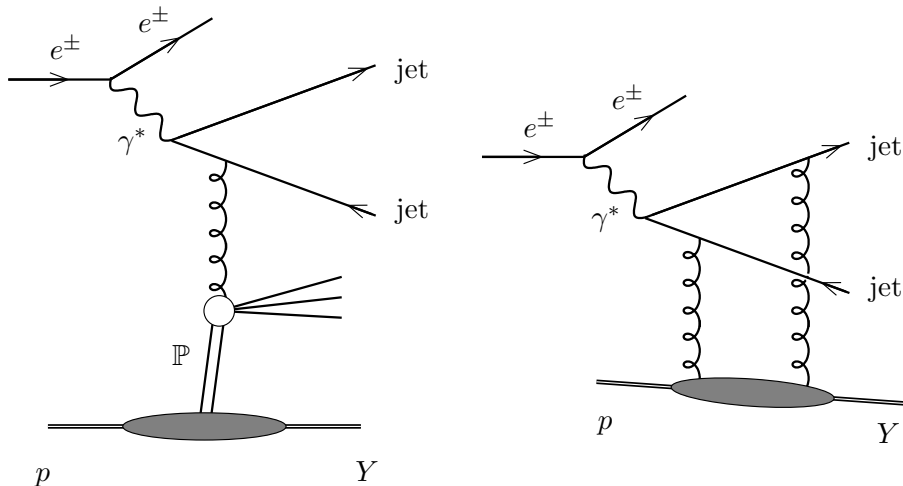


Fig. 1. Resolved (left panel) and direct Pomeron (right panel) contributions to two jets production.

For moderate invariant mass M^2 of the diffractively produced state X , such a state can be modeled in perturbation theory by a $q\bar{q}$ pair, or by higher Fock states as a $q\bar{q}g$ state for larger values of M^2 . Based on such a model, with a two-gluon exchange picture for the Pomeron, a good description of HERA data for diffraction [3, 4] could be achieved [5]. In the direct components considered there, the $q\bar{q}g$ diffractive state has been studied in two particular limits. The first one, valid for very large Q^2 , corresponds to a collinear approximation in which the transverse momentum of the gluon is assumed to be much smaller than the transverse momentum of the emitter [6]. The second one [7, 8], valid for very large M^2 , is based on the assumption of a strong ordering of longitudinal momenta, encountered in BFKL equation [9]. Both these approaches were combined in order to describe HERA data for DDIS [10].

It would be natural to extend the HERA studies to similar hard diffractive events at LHC. The idea here is to adapt the concept of photoproduction of diffractive jets, which was performed at HERA [11, 12], now with a flux of quasi-real photons in ultraperipheral collisions (UPC) [13], relying on the notion of equivalent photon approximation. In both cases, the hard scale is provided by the invariant mass of the tagged jets.

We here report on our computation [14] of the $\gamma^* \rightarrow q\bar{q}g$ impact factor at tree level with an arbitrary number of t -channel gluons described within the Wilson line formalism, also called QCD shockwave approach [15]. As an aside, we rederive the $\gamma^* \rightarrow q\bar{q}$ impact factor. In particular, the $\gamma^* \rightarrow q\bar{q}g$

transition is computed without any soft or collinear approximation for the emitted gluon, in contrast with the above mentioned calculations. These results provide a necessary generalization of building blocks for inclusive DDIS (of potential significant phenomenological importance [16]) as well as for two- and three-jet diffractive production.

2. The shockwave formalism in a nutshell

Balitsky's shockwave formalism is very powerful in determining evolution equations and impact factors at next-to-leading order for inclusive processes [17], at semi-inclusive level for p_t -broadening in pA collisions [18] or in the evaluation of the triple Pomeron vertex beyond the planar limit [19], when compared with usual methods based on summation of contributions of individual Feynman diagrams computed in momentum space. It is an effective way of estimating the effect of multigluon exchange, formulated in coordinate space and thus natural in view of describing saturation [20].

We introduce the light cone vectors n_1 and n_2

$$n_1 = (1, 0, 0, 1), \quad n_2 = \frac{1}{2} (1, 0, 0, -1), \quad n_1^+ = n_2^- = n_1 \cdot n_2 = 1, \quad (1)$$

and the Wilson lines as

$$U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp \left[ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) dz_i^+ \right]. \quad (2)$$

The operator b_η^- is the external shock-wave field built from slow gluons whose momenta are limited by the longitudinal cut-off defined by the rapidity η

$$b_\eta^- = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot z} b^-(p) \theta \left(e^\eta - \frac{|p^+|}{P^+} \right), \quad (3)$$

where P^+ is the typical large + momentum of the problem, to be identified with p_γ^+ later on. We will denote the longitudinal cut-off $\sigma = e^\eta P^+ = \alpha P^+$.

We use the light cone gauge $\mathcal{A} \cdot n_2 = 0$, with \mathcal{A} being the sum of the external field b and the quantum field A

$$\mathcal{A}^\mu = A^\mu + b^\mu, \quad b^\mu(z) = b^-(z^+, \vec{z}) n_2^\mu = \delta(z^+) B(\vec{z}) n_2^\mu, \quad (4)$$

where $B(\vec{z})$ is a profile function. Indeed, let us consider an external gluon

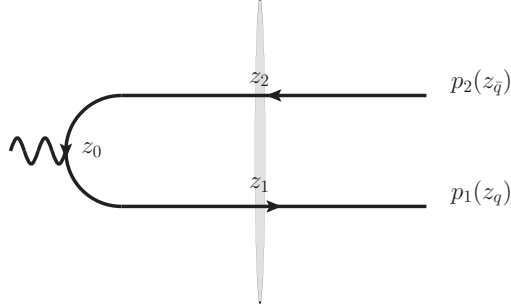


Fig. 2. Diagram contributing to the impact factor for two jet production

field b^μ in its rest frame and boost it along the $+$ direction. One obtains :

$$\begin{aligned} b^+(x^+, x^-, \vec{x}) &\rightarrow \frac{1}{\lambda} b^+ \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right), \\ b^-(x^+, x^-, \vec{x}) &\rightarrow \lambda b^- \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right), \\ b^i(x^+, x^-, \vec{x}) &\rightarrow b^i \left(\lambda x^+, \frac{1}{\lambda} x^-, \vec{x} \right). \end{aligned}$$

Assuming that the field vanishes at infinity, one immediately gets that only its minus component survives the boost in the limit $\lambda \rightarrow \infty$, and that it does not depend on x^- and contains $\delta(x^+)$, thus justifying the form of b^μ in Eq. (4).

We use intensively in the following the dipole operator constructed from the Wilson line (2), namely $\mathbf{U}_{12} = \frac{1}{N_c} \text{tr} \left(\mathbf{U}_1 \mathbf{U}_2^\dagger \right) - 1$.

3. Impact factor for $\gamma \rightarrow q\bar{q}$ transition

At leading order, the diagram contributing to the impact factor for $\gamma \rightarrow q\bar{q}$ transition is shown in Fig. 2, in which z 's denote the coordinates of interaction points with the photon and the shock wave. After projection on the color singlet state and subtraction of the contribution without interaction with the shock wave, the contribution of this diagram can be written in the momentum space as (factorizing out a global QED factor $-ie_q$)

$$M_0^\alpha = N_c \int d\vec{z}_1 d\vec{z}_2 F(p_q, p_{\bar{q}}, z_0, \vec{z}_1, \vec{z}_2)^\alpha \mathbf{U}_{12}. \quad (5)$$

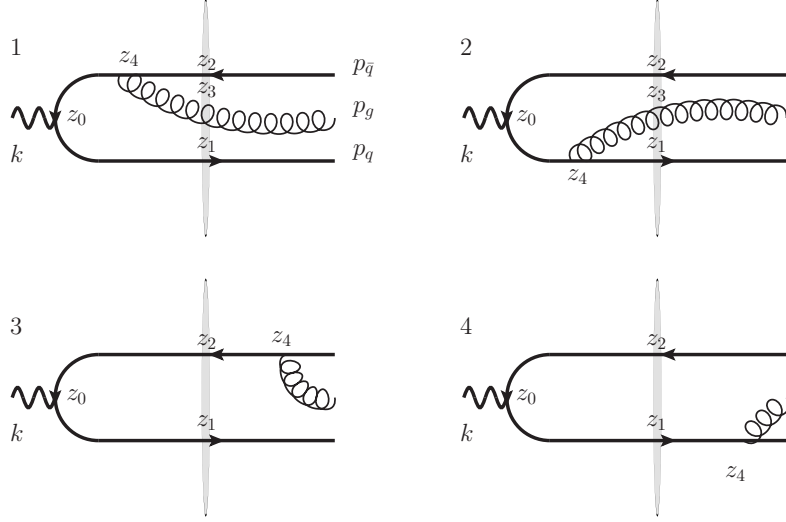


Fig. 3. Diagrams contributing to the impact factor for three jet production

Denoting $Z_{12} = \sqrt{x_q x_{\bar{q}} \vec{z}_{12}^2}$, we get for a longitudinally polarized photon

$$F(p_q, p_{\bar{q}}, k, \vec{z}_1, \vec{z}_2)^\alpha \varepsilon_{L\alpha} = \theta(p_q^+) \theta(p_{\bar{q}}^+) \frac{\delta(k^+ - p_q^+ - p_{\bar{q}}^+)}{(2\pi)^2} e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2} \\ \times (-2i) \delta_{\lambda_q, -\lambda_{\bar{q}}} x_q x_{\bar{q}} Q K_0(Q Z_{12}), \quad (6)$$

whereas for a transversally polarized photon

$$F(p_q, p_{\bar{q}}, k, \vec{z}_1, \vec{z}_2)^j \varepsilon_{Tj} = \theta(p_q^+) \theta(p_{\bar{q}}^+) \frac{\delta(k^+ - p_q^+ - p_{\bar{q}}^+)}{(2\pi)^2} e^{-i\vec{p}_q \cdot \vec{z}_1 - i\vec{p}_{\bar{q}} \cdot \vec{z}_2} \\ \times \delta_{\lambda_q, -\lambda_{\bar{q}}} (x_q - x_{\bar{q}} + s\lambda_q) \frac{\vec{z}_{12} \cdot \vec{\varepsilon}_T}{\vec{z}_{12}^2} Q Z_{12} K_1(Q Z_{12}). \quad (7)$$

4. Impact factor for $\gamma \rightarrow q\bar{q}g$ transition

In the case of the $q\bar{q}g$ Fock final state the contributing diagrams are shown in Fig. 3. After projection on the color singlet state and subtraction of the contribution without interaction with the shock wave, the result can

be put in the form

$$\begin{aligned}
M^\alpha &= N_c^2 \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha \\
&\times \frac{1}{2} (\mathbf{U}_{32} + \mathbf{U}_{13} - \mathbf{U}_{12} + \mathbf{U}_{32} \mathbf{U}_{13}) \\
&+ N_c \int d\vec{z}_1 d\vec{z}_2 F_2(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^\alpha \frac{N_c^2 - 1}{2N_c} \mathbf{U}_{12}. \quad (8)
\end{aligned}$$

In this equation, the first two lines and the third one correspond to contributions to the impact factor, respectively, of the diagrams 1 and 2 of Fig. 3 and of the diagrams 3 and 4 of it. The explicit expressions for the functions F_i , for both longitudinally and transversally polarized photon can be found in ref. [14].

5. The 2- and 3-gluon approximation

We first notice that the dipole operator \mathbf{U}_{ij} involves terms at least of order g^2 . Hence for only two or three exchanged gluons one can neglect the quadrupole term in the amplitude M^α which results in the simpler expression

$$\begin{aligned}
M^\alpha &\stackrel{\text{g}^3}{=} \frac{1}{2} \int d\vec{z}_1 d\vec{z}_2 \mathbf{U}_{12} \left[(N_c^2 - 1) \tilde{F}_2(\vec{z}_1, \vec{z}_2)^\alpha \right. \\
&\left. + \int d\vec{z}_3 \{ N_c^2 F_1(\vec{z}_1, \vec{z}_3, \vec{z}_2)^\alpha + N_c^2 F_1(\vec{z}_3, \vec{z}_2, \vec{z}_1)^\alpha - F_1(\vec{z}_1, \vec{z}_2, \vec{z}_3)^\alpha \} \right]. \quad (9)
\end{aligned}$$

Those integrals can be performed analytically when $\vec{p}_q = \vec{p}_g = \vec{p}_{\bar{q}} = \vec{0}$. They are otherwise expressible as a simple convergent integral over the interval $[0, 1]$.

6. Towards the next-to-leading-order corrections

The virtual corrections to the $\gamma^* \rightarrow q\bar{q}$ involve two kinds of contributions. The diagrams contributing to virtual corrections in which the radiated gluon does not cross the shock wave are shown in Fig. 4, and the diagrams in which the radiated gluon interacts with the shock wave are illustrated in the Fig. 5. One should note that although these virtual corrections only involve one-loop diagrams, the complications arise due to the presence of many different scales. Indeed, our aim is to obtain results in the general kinematics where the virtuality of incoming photon, the t -channel momentum transfer and the invariant mass M^2 of the diffractive two-jet state are arbitrary.

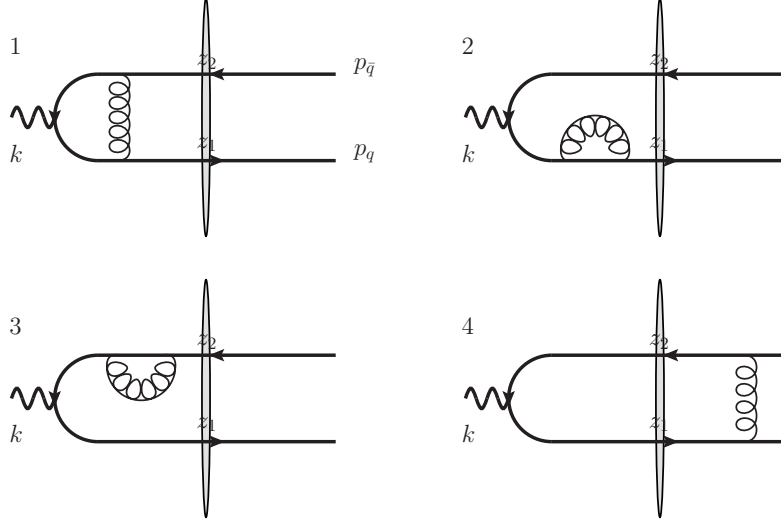


Fig. 4. Diagrams contributing to virtual corrections in which the radiated gluon doesn't cross the shock wave.

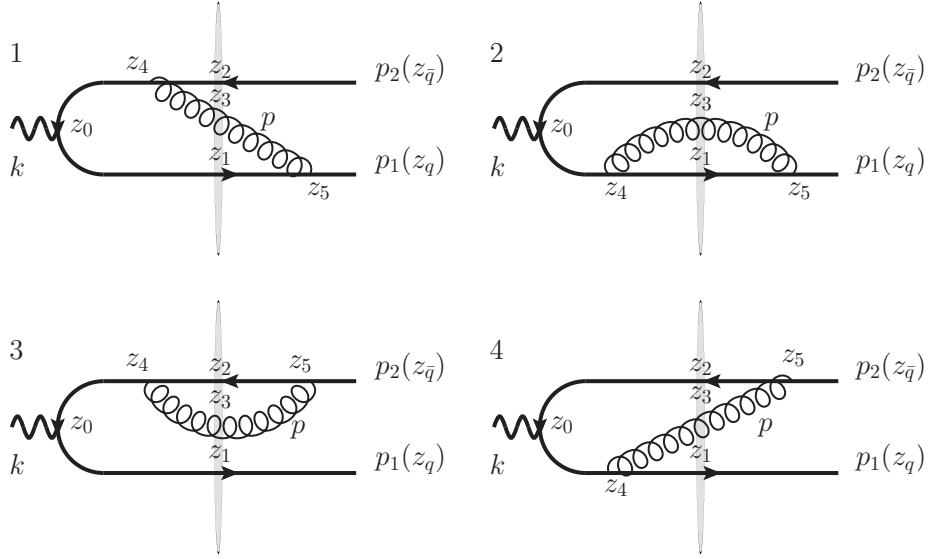


Fig. 5. Diagrams contributing to virtual corrections in which the radiated gluon interacts with the shock wave.

Additionally, this impact factor is a function of the virtuality of t -channel exchanged gluons.

We now provide some intermediate results of our computation. First, we present the matrix element corresponding to diagrams 1, 2, 3 of Fig. 4 [21]. We work in dimensional regularization for the transverse momentum space, i.e. $d = D - 2 = 2 + 2\epsilon$, and introduce the regularization scale μ , and the related dimensionless scale $\tilde{\mu}^2 = \mu^2/Q^2$. Denoting $p_{ij} \equiv p_i - p_j$, we introduce $p_\perp = p_{q1\perp}$, $\vec{p}^2 = -p_\perp^2$ and $w = \vec{p}^2/Q^2$. For simplicity, we write $x = x_q$. We get for the case of a longitudinally polarized photon

$$\begin{aligned}
T_{fi}|_{\epsilon_\alpha=n_{2\alpha}} &= -ig^2 \frac{N_c^2 - 1}{2N_c} \text{tr}(U(p_{1\perp})U^\dagger(-p_{2\perp}))\delta(p_{q1\perp} - p_{\gamma\perp} + p_{\bar{q}2\perp}) \\
&\times \delta(p_q^+ - p_\gamma^+ + p_{\bar{q}}^+)\theta(p_q^+)\theta(p_{\bar{q}}^+) \\
&\times \frac{\Gamma(1-\epsilon)}{(16\pi^3)^{1+\epsilon}} \frac{1}{\sqrt{2p_\gamma^+}\sqrt{2p_q^+}\sqrt{2p_{\bar{q}}^+}} \frac{x(1-x)p_\gamma^+\bar{u}_{p_q}\gamma^+v_{p_{\bar{q}}}}{x(1-x)Q^2 + \vec{p}^2} \\
&\times \left(\left(2\ln\left(\frac{(1-x)x}{\alpha^2}\right) - 3 \right) \left(\ln\left(\frac{(w-x^2+x)^2}{(1-x)x\tilde{\mu}^2}\right) + \frac{1}{\epsilon} \right) \right. \\
&\quad \left. + \ln^2\left(\frac{x}{1-x}\right) - \frac{\pi^2}{3} + 6 \right). \tag{10}
\end{aligned}$$

Expanding the photon momentum in the Sudakov basis (1) as

$$p_\gamma = p_\gamma^+ n_1 - \frac{Q^2}{2p_\gamma^+} n_2 \tag{11}$$

one can explicitly check the electromagnetic gauge invariance for this group of diagrams since

$$T_{fi}|_{\epsilon_\alpha=n_{1\alpha}} = \frac{Q^2}{2p_\gamma^{+2}} T_{fi}|_{\epsilon_\alpha=n_{2\alpha}}. \tag{12}$$

Similarly, for the case of a transversally polarized photon, one gets

$$\begin{aligned}
T_{fi}|_{transverse} &= -ig^2 \frac{N_c^2 - 1}{2N_c} \text{tr}(U(p_{1\perp})U^\dagger(-p_{2\perp}))\delta(p_{q1\perp} - p_{\gamma\perp} + p_{\bar{q}2\perp}) \\
&\times \delta(p_q^+ - p_\gamma^+ + p_{\bar{q}}^+)\theta(p_q^+)\theta(p_{\bar{q}}^+) \\
&\times \frac{\Gamma(1-\epsilon)}{(16\pi^3)^{1+\epsilon}} \frac{\epsilon_i}{\sqrt{2p_\gamma^+}\sqrt{2p_q^+}\sqrt{2p_{\bar{q}}^+}} - \frac{(\frac{1}{2}\bar{u}_{p_q}[\gamma^i\hat{p}_\perp]\gamma^+v_{p_{\bar{q}}} + (2x-1)p^i\bar{u}_{p_q}\gamma^+v_{p_{\bar{q}}})}{2(x(1-x)Q^2 + \vec{p}^2)} \\
&\times \left[\left(2\ln\left(\frac{(1-x)x}{\alpha^2}\right) - 3 \right) \left(\ln\left(\frac{w-x^2+x}{\tilde{\mu}^2}\right) + \frac{(1-x)x\ln\left(\frac{(1-x)x}{w-x^2+x}\right)}{w} + \frac{1}{\epsilon} \right) \right. \\
&\quad \left. + \ln^2\left(\frac{x}{1-x}\right) - \frac{\pi^2}{3} + 6 \right]. \tag{13}
\end{aligned}$$

Second, we present the singular part of diagram 4 of fig. 4 involving final state interaction. The result for a longitudinally polarized photon reads

$$\begin{aligned}
T_{fi}|_{\epsilon_\alpha=n_{2\alpha}} &= i \frac{N_c^2 - 1}{2N_c} \text{tr}(U(p_{1\perp})U^\dagger(-p_{2\perp})) \delta(p_{\gamma\perp} - p_{1q\perp} - p_{2\bar{q}\perp}) \\
&\times \delta(p_\gamma^+ - p_q^+ - p_{\bar{q}}^+) \frac{\Gamma(1-\varepsilon)}{(16\pi^3)^{1+\varepsilon}} \frac{1}{\sqrt{2p_\gamma^+} \sqrt{2p_q^+} \sqrt{2p_{\bar{q}}^+}} \\
&\times \left\{ \frac{(1-x)x \bar{u}_{p_q} \gamma^+ v_{p_{\bar{q}}} p_\gamma^+}{\vec{p}^2 + Q^2(1-x)x} \left[\ln^2 \left(\frac{(1-x)x}{\alpha^2} \right) - \ln^2 \left(\frac{1-x}{x} \right) \right. \right. \\
&\quad \left. \left. + 2 \ln \left(\frac{(1-x)x}{\alpha^2} \right) \left(\ln \left(\frac{(\vec{p}^2 + Q^2(1-x)x)^2}{Q^2(x\vec{p}_{\bar{q}} - (1-x)\vec{p}_q)^2} \right) + i\pi \right) \right] + C_{\parallel}^{fs} \right\}, \quad (14)
\end{aligned}$$

while for a transversally polarized photon we obtain

$$\begin{aligned}
T_{fi}|_{\text{transverse}} &= i \frac{N_c^2 - 1}{2N_c} \text{tr}(U(p_{1\perp})U^\dagger(-p_{2\perp})) \delta(p_{\gamma\perp} - p_{1q\perp} - p_{2\bar{q}\perp}) \\
&\times \delta(p_\gamma^+ - p_q^+ - p_{\bar{q}}^+) \frac{\Gamma(1-\varepsilon)}{(16\pi^3)^{1+\varepsilon}} \frac{\epsilon_i}{\sqrt{2p_\gamma^+} \sqrt{2p_q^+} \sqrt{2p_{\bar{q}}^+}} \\
&\times \left\{ - \frac{(2x-1)p_\perp^i \bar{u}_{p_q} \gamma^+ v_{p_{\bar{q}}} + \frac{1}{2} \bar{u}_{p_q} \gamma^+ [\gamma_\perp^i \hat{p}_\perp] v_{p_{\bar{q}}}}{(Q^2(1-x)x + \vec{p}^2)} \left[\frac{1}{2} \ln^2 \left(\frac{(1-x)x}{\alpha^2} \right) \right. \right. \\
&\quad \left. \left. - \frac{1}{2} \ln^2 \left(\frac{x}{1-x} \right) + \ln \left(\frac{(1-x)x}{\alpha^2} \right) \left(\frac{Q^2(1-x)x}{\vec{p}^2} \ln \left(\frac{Q^2(1-x)x}{Q^2(1-x)x + \vec{p}^2} \right) \right. \right. \right. \\
&\quad \left. \left. \left. + \ln \left(\frac{(1-x)x (Q^2(1-x)x + \vec{p}^2)}{(x\vec{p}_{\bar{q}} - (1-x)\vec{p}_q)^2} \right) + i\pi \right) \right] + C_{\perp}^{fs} \right\}. \quad (15)
\end{aligned}$$

In eqs. (14,15), C_{\parallel}^{fs} and C_{\perp}^{fs} are finite terms which are too lengthy to be written here.

7. Conclusion

Dijet production in DDIS at HERA was recently analyzed [22]. A precise comparison of dijet versus triple-jet production, which has not been performed yet at HERA [23], would be of much interest. Investigations of the azimuthal distribution of dijets in diffractive photoproduction performed by ZEUS [24] show sign of a possible need for a 2-gluon exchange model, which is part of the shock-wave mechanism. Our calculation could be used for phenomenological studies of those experimental results. Complementary

studies could be performed at LHC with UPC events. A full quantitative first principle analysis of this will be possible after completing our program of computing virtual corrections to the $\gamma^* \rightarrow q\bar{q}$ impact factor [25], for which we have provided here intermediate results.

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